Modified Newton's Law of Gravitation Due to Minimal Length in Quantum Gravity

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Abstract

A recent theory about the origin of the gravity suggests that the gravity is originally an entropic force. In this work, we discuss the effects of generalized uncertainty principle (GUP) which is proposed by some approaches to *quantum gravity* such as string theory, black hole physics and doubly special relativity theories (DSR), on the area law of the entropy. This leads to a $\sqrt{\text{Area}}$-type correction to the area law of entropy which imply that the number of bits N is modified. Therefore, we obtain a modified Newton's law of gravitation. Surprisingly, this modification agrees with different sign with the prediction of Randall-Sundrum II model which contains one uncompactified extra dimension. Furthermore, such modification may have observable consequences at length scales much larger than the Planck scale.

1 Introduction

The earliest idea about the connection between gravitation and the existence of a fundamental length was proposed in [1]. In the last two decades, the existence of a minimal length is one of the most interesting predictions of some approaches related to quantum gravity such as String Theory, Black hole physics and non-commutative geometry [2–4]. The existence of a minimal length is considered as a consequence of the string theory because strings obviously can not interact at distances smaller than the string size. Furthermore, the black hole physics suggests that the uncertainty relation should be modified near the Planck energy scale due to the fact that the photons emitted from the black hole suffers from two major errors; the first one is the error by Heisenberg classical analysis and the second one is because the black hole mass varies during the emission process and the radius of the horizon changes accordingly [2,3]. An interesting measure gedanken experiment was proposed in [5] involving micro-black holes at the Planck scale of spacetime which leads to the GUP. This independent model depends on Heisenberg principle and Schwarzschild radius. Recently, It was found that polymer quantization suggests the existence of minimal length in similar way to string theory and black hole physics [6]. Therefore, all these different approaches suggests that the standard uncertainty relation in quantum mechanics

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is modified to yield *Generalized Uncertainty Principle* (GUP) [2,3,6]. In light of this, such modifications can play an essential role as the quantum gravitational corrections which would open an interesting window for quantum gravity phenomenology [7–18].

In a one-dimensional chain as the Ising model, when assuming that every single spin is positioned at a distance d apart from the two neighborhoods. Then, the macroscopic state of such a chain is defined by d. The entire chain would have various configurations so that if $d \to l$, the chain has much less configurations than if $d \ll l$, where l is the chain's length. Statistically, the entropy is given by the number of microscopic states $S = k_B \ln \Omega$. Due to second law of thermodynamics, such a system tends to approach a state of maximal entropy so that the chain in the macroscopic state d tends to go to a d state with a much higher entropy. The force that causes such a statistical tendency is defined as the entropic force. In light of this, the entropic force is a phenomenological mechanism deriving a system to approach maximum entropy i.e., increasing the number of microscopic states that will be inhered in the system's phase space. There are various examples on the entropic force, for example polymer molecules and the elasticity of rubber bands.

Recently, Verlinde proposed that the gravity is not fundamental force and can be considered as an entropic force [19]. The earliest idea about gravity that is regarded as a non-fundamental interaction has been introduced by Sakharov [20], where the spacetime background is assumed to emerge as a mean field approximation of underlying microscopic degrees of freedom. Similar behavior is observed in hydrodynamics [21]. It is found that the entropy of black hole is related to the horizon's area at the black hole's horizon, while the temperature is related to the surface gravity. Both entropy and temperature are assumed to be related to the mass of black hole [22]. Thus, the connection between thermodynamics and geometry leads to Einstein's equations of gravitational field from relations connecting heat, entropy, and temperature [23]. The Einstein's equations connect energy-momentum tensor with space geometry. Advocating the gravity as non-fundamental interaction leads to the assumption that gravity would be explained as an entropic force caused by changes in the information associated with the positions of material bodies [19]. When combining the entropic force with the Unruh temperature, the second law of Newton is obtained. But when combining it with the holographic principle and using the equipartition law of energy, the Newton's law of gravitation is obtained. It was investigated in [24] modification of the entropic force due to corrections to the area law of entropy which is derived from quantum effects and extra dimensions.

Apart from the controversial debate on the origin of gravity [25, 26], we investigate the impact of GUP on the entropic force and derive essential quantities including potential modification to the Newton's law of gravity.

There were some studies for the effect of some versions of GUP on the Newton's law of gravity in [27]. Also, Non-Commutative Geometry which is considered as a completely planck scale effect has been studied to derive the modified Newton's law of gravity [28–31]. All these approaches for studying the Planck scale effects on the Newton's law of gravity are based on the following scheme: modified theory of gravity → modified black hole entropy→ modified holographic surface entropy → Newton's law corrections. We followed the same scheme in our paper using the new version of GUP proposed in [32–34], and we a got a new corrections in our current work, which are distinct from the previous studies. Moreover, we compared our results with Randall-Sundrum model of extra dimension which

also predicts the modification of Newton's law of gravity at the planck scale [35, 36], where we think there may be some connection between generalized uncertainty principle and extra dimension theories because they predicting similar physics at least for the case of Newton's law of gravity which may be considered as a distinct result from the previous studies.

The present paper is organized as follows. Section 2 reviews briefly the generalized uncertainty principle that was proposed in [32–34] . Section 3 is devoted to review the entropic force and gravitational interaction [19]. The effect of utilizing GUP impact on the entropic force is introduced in section 4. In section 5, we estimate the GUP correction to the Newton's law of gravitation. The conclusions are given in section 6.

2   The Generalized Uncertainty Principle

It is conjectured that the standard commutation relations at short distances would be modified. A new form of GUP was proposed [32–34] and found consistent with the Doubly Special Relativity (DSR) theories, the string theory and the black holes physics. It predicts a maximum observable momentum and a minimal measurable length. With satisfying Jacobi identity, GUP is found to ensure the relations $[x_i, x_j] = 0 = [p_i, p_j]$:

$$[x_i, p_j] = i\hbar \left[\delta_{ij} - \alpha \left(p\delta_{ij} + \frac{p_i p_j}{p} \right) + \alpha^2 \left(p^2 \delta_{ij} + 3p_i p_j \right) \right],$$ (2.1)

where $\alpha = \alpha_0/M_p c = \alpha_0 \ell_p/\hbar$ and $M_p c^2$ stand for Planck energy. M_p and ℓ_p is Planck mass and length, respectively. α_0 sets on the upper and lower bounds to α.

For a particle having an energy scale comparable to the Planck's one, the physical momentum would be a subject of a modification [32–34]

$$p_i = p_{0i} \left(1 - \alpha p_0 + 2\alpha^2 p_0^2 \right),$$ (2.2)

where $x_i = x_{0i}$ and p_{0j} satisfy the canonical commutation relations $[x_{0i}, p_{0j}] = i\hbar \, \delta_{ij}$. Here, p_{0i} can be interpreted as the momentum at low energies and p_i as that at high energies, and the variable p_0 is the value of the momentum at the low energy scales.

This newly proposed GUP suggests that the space is quantized into fundamental units which may be the Planck length. The quantization of space has been shown within the context of loop quantum gravity in [37].

In a series of earlier papers, the effects of GUP was investigated on atomic, condensed matter, preheating phase of the universe systems, black holes at LHC [38–40], the weak equivalence principle (WEP), the Liouville theorem (LT) in statistical mechanics [41]. It was found that the GUP can potentially explain the small observed violations of the WEP in neutron interferometry experiments [42] and also predicts a modified invariant phase space which is relevant to the Liouville theorem. It was derived in [38] the first bound for α_0 is about $\sim 10^{17}$, which would approximately gives $\alpha \sim 10^{-2}$ GeV^{-1}. The other bound

of α_0 which is $\sim 10^{10}$. This bound means that $\alpha \sim 10^{-9}$ GeV^{-1}. As discussed in [43], the exact bound on α can be obtained by comparison with observations and experiments. It seems that the gamma rays burst would allow us to set an upper value for the GUP-charactering parameter α which we would like to report on this in the future.

Recently, it has been suggested in [44] that the GUP implications can be measured directly in quantum optics laboratories which definitely confirm the theoretical predictions given in [13,38]. Definitely, this is considered as a milestone in the road of quantum gravity phenomenology.

In section 3, we briefly review the assumptions that the gravitational force would be originated from an entropic nature.

3 Gravity as an entropic force

Recently, Erick Verlinde [19] has utilized Sakharov's proposal [20] that the gravity would not be considered a fundamental force. Concretely, it was suggested that the gravitational force might be originated to an entropic nature. As discussed in the introduction, this assumption is based on the relation between the gravitation and thermodynamics [22]. According to thermodynamics and holographic principle, Verlinde's approach results in the Newton's law. Moreover, the Friedmann equations can also be derived [45]. At temperature T, the entropic force F of a gravitational system is given as

$$F\Delta x = T\Delta S, \tag{3.1}$$

where ΔS is the change in the entropy so that at a displacement Δx, each particle carries its own portion of entropy change. From the correspondence between the entropy change ΔS and the information about the boundary of the system and using Bekenstein's argument [22], it is assumed that $\Delta S = 2\pi k_B$, where $\Delta x = \hbar/m$ and k_B is the Boltzmann constant.

$$\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x, \tag{3.2}$$

where m is the mass of the elementary component c is speed of light and $\hbar$ is the Planck constant, respectively.

Turning to the holographic principle which assumes that for any closed surface without worrying about its geometry inside, all physics can be represented by the degrees of freedom on this surface itself. This implies that the quantum gravity can be described by a topological quantum field theory, for which all physical degrees of freedom can be projected onto the boundary [46]. The information about the holographic system is given by N bits forming an ideal gas. It is conjectured that N is proportional to the entropy of the holographic screen,

$$N = \frac{4S}{k_B}, \tag{3.3}$$

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then according to Bekenstein's entropy-area relation [22]

$$S = \frac{k_B c^3}{4G\hbar} A. \tag{3.4}$$

Therefore, one gets

$$N = \frac{Ac^3}{G\hbar} = \frac{4\pi r^2 c^3}{G\hbar}, \tag{3.5}$$

where r is the radius of the gravitational system and the area of the holographic screen $A = 4\pi r^2$ is implemented in deriving this equation. It is assumed that each bit emerges outwards from the holographic screen i.e., one dimension. Therefore each bit carries an energy equal to $k_B T/2$, so using the equipartition rule to calculate the energy of the system, one gets

$$E = \frac{1}{2} N k_B T = \frac{2\pi c^3 r^2}{G\hbar} k_B T = Mc^2. \tag{3.6}$$

By substituting Eq. (3.1) and Eq. (3.2) into Eq. (3.6), we get

$$F = G\frac{Mm}{r^2}, \tag{3.7}$$

making it clear that Newton's law of gravitation can be derived from first principles.

In section 4, we study the effect of GUP-approach on the entropic force and hence on the Newton's law of gravitation.

4   GUP impact on the black hole thermodynamics

Taking into consideration the GUP-approach [32–34], and because black holes are considered as a good laboratories for the clear connection between thermodynamics and gravity, the black hole thermodynamics will be analyzed in this section. Furthermore, how the entropy would be affected shall be investigated, as well. In Hawking radiation, the emitted particles are mostly photons and standard model (SM) particles. From kinetic theory of gases, let us assume that gatherings or clouds of points in the velocity space are equally spread in all directions. There is no reason that particles would prefer to be moving in a certain direction. Then, the three-moments are simply equal

$$p_1 \approx p_2 \approx p_3, \tag{4.1}$$

leading to

$$\begin{aligned} p^2 &= \sum_{i=1}^{3} p_i p_i \approx 3\, p_i^2, \\ \langle p_i^2 \rangle &\approx \frac{1}{3}\, \langle p^2 \rangle. \end{aligned} \tag{4.2}$$

In order to find a relation between $\langle p^2 \rangle$ and Δp^2, we assume that the black hole behaves like a black body, while it emits photons. Therefore, from Wien's law, the temperature corresponding to the peak emission is given by

$$c \langle p \rangle = 2.82 \, T_H. \tag{4.3}$$

We should keep in mind that the numerical factor, 2.82, should be modified by the grey-body factors which arise due to the spacetime curvature around the black hole, but for simplicity we are just ignoring this modification.

From Hawking uncertainty proposed by Scardigli in [9] and Adler *et al.* [47], the Hawking's temperature reads

$$T_H = \frac{1}{\pi} c \, \Delta p = \frac{1}{2.82} c \langle p \rangle. \tag{4.4}$$

With the relation $\langle p^2 \rangle = \Delta p^2 + \langle p \rangle^2$, we get

$$
\begin{aligned}
\langle p \rangle &= 2.82 \frac{1}{\pi} \Delta p = \sqrt{\mu} \, \Delta p, \\
\langle p^2 \rangle &= (1 + \mu) \, \Delta p^2,
\end{aligned}
\tag{4.5}
$$

where $\mu = (2.82/\pi)^2$. Again, the parameter μ is modified if we consider the grey-body factors which arise due to the spacetime curvature around the black hole.

In order to have a corresponding inequality for Eq. (2.1), we can utilize the arguments given in [48]. Then

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left[1 - \alpha \langle p \rangle - \alpha \langle \frac{p_i^2}{p} \rangle + \alpha^2 \langle p^2 \rangle + 3\alpha^2 \langle p_i^2 \rangle \right]. \tag{4.6}$$

It is apparent that implementing the arguments given in Eqs. (4.2) and (4.5) in the inequality gievn in Eq. (4.6) leads to

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left[1 - \alpha_0 \, \ell_p \left(\frac{4}{3} \right) \, \sqrt{\mu} \, \frac{\Delta p}{\hbar} + 2 \, (1 + \mu) \, \alpha_0^2 \, \ell_p^2 \, \frac{\Delta p^2}{\hbar^2} \right], \tag{4.7}$$

The resulting inequality, Eq. (4.7), is the only one that follows from Eq. (2.1). Solving it as a quadratic equation in Δp results in

$$\frac{\Delta p}{\hbar} \geq \frac{2\Delta x + \alpha_0 \, \ell_p \left(\frac{4}{3} \sqrt{\mu} \right)}{4 \, (1 + \mu) \, \alpha_0^2 \, \ell_p^2} \left(1 - \sqrt{1 - \frac{8 \, (1 + \mu) \, \alpha_0^2 \ell_p^2}{\left(2\Delta x + \alpha_0 \ell_p \left(\frac{4}{3} \right) \, \sqrt{\mu} \right)^2}} \right). \tag{4.8}$$

The negatively-signed solution is considered as the one that refers to the standard uncertainty relation as $\ell_p / \Delta x \to 0$. Using the Taylor expansion, we obviously find that

$$\Delta p \geq \frac{1}{\Delta x} \left(1 - \frac{2}{3} \alpha_0 \ell_p \sqrt{\mu} \frac{1}{\Delta x} \right). \tag{4.9}$$

Because the energy change reads $\Delta E \approx c \, \Delta p$ and according to Scardigli in [9] and Adler *et al.* [47], one can define the uncertainty in the energy ΔE as the energy carried away from the black hole through the emitted photon. In the following, we implement

the procedure introduced in [49]. We utilize the GUP-approach [32–34] together with the assumptions given in Eqs. (4.2) and (4.5). Then, using natural units that $\hbar = c = 1$

$$\Delta E \geq \frac{1}{\Delta x}\left(1 - \frac{2}{3}\alpha_0\ell_p\sqrt{\mu}\frac{1}{\Delta x}\right). \tag{4.10}$$

For a black hole absorbing a quantum particle with energy E and size R, the area of the black hole is supposed to increase by the amount [50].

$$\Delta A \geq 8\pi\,\ell_p^2\,E\,R, \tag{4.11}$$

The quantum particle itself implies the existence of finite bound given by

$$\Delta A_{min} \geq 8\pi\,\ell_p^2\,E\,\Delta\,x. \tag{4.12}$$

Using Eq. (4.10) in the inequality (4.12), we obtain,

$$\Delta A_{min} \geq 8\pi\ell_p^2\left[1 - \frac{2}{3}\alpha_0\ell_p\sqrt{\mu}\frac{1}{\Delta x}\right]. \tag{4.13}$$

According to the argument given in [49], the length scale is chosen to be the inverse surface gravity

$$\Delta x = 2\,r_s, \tag{4.14}$$

where r_s is the is the Schwarzschild radius. This argument implies that

$$(\Delta x)^2 \sim \frac{A}{\pi}. \tag{4.15}$$

Substituting Eq. (4.15) into Eq. (4.13), we got

$$\Delta A_{min} = \lambda\ell_p^2\left[1 - \frac{2}{3}\,\alpha_0\,\ell_p\,\sqrt{\frac{\mu\,\pi}{A}}\right], \tag{4.16}$$

where parameter λ will be fixed later. According to [22], the black hole's entropy is conjectured to depend on the horizon's area. From the information theory [51], it has been found that the minimal increase of entropy should be independent on the area. It is just one bit of information which is $b = \ln(2)$.

$$\frac{dS}{dA} = \frac{\Delta S_{min}}{\Delta A_{min}} = \frac{b}{\lambda\ell_p^2\left[1 - \frac{2}{3}\,\alpha_0\,\ell_p\,\sqrt{\mu\,\pi\,A}\right]}, \tag{4.17}$$

where b is a parameter. By expanding the last expression in orders of α and then integrating it, we get the entropy

$$S = \frac{b}{\lambda\ell_p^2}\left[A + \frac{4}{3}\,\alpha_0\,\ell_p\,\sqrt{\mu\,\pi\,A}\right]. \tag{4.18}$$

Using Hawking-Bekenstein assumption, which relates entropy with the area, the value of constants $b/\lambda = 1/4$, so that

$$S = \frac{A}{4\,\ell_p^2} + \frac{2}{3}\,\alpha_0\,\sqrt{\pi\,\mu\,\frac{A}{4\,\ell_p^2}}. \tag{4.19}$$

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Although it was found in [52] that the power–law corrections to Bekenstein–Hawking areaentropy are ruled out based on arguments from Boltzmann–Einstein formula, but the it was found that the power-law corrections may explain the observed cosmic acceleration today [53].

We conclude that the entropy is directly related to the area and gets a correction when applying GUP-approach. The temperature of the black hole is

$$T = \frac{\kappa}{8\pi}\frac{dA}{dS} = \frac{\kappa}{8\pi}\left[1 - \frac{2}{3}\,\alpha_0\,\ell_p\,\sqrt{\mu\,\frac{\pi}{A}}\,\right]. \tag{4.20}$$

So far, we conclude that the temperature is not only proportional to the surface gravity but also it depends on the black hole's area.

5 Modified Newton's law of gravitation due to GUP

In this section we study the implications of the corrections calculated for the entropy in Eq. (4.19), and calculate how the number of bits of Eq. (3.3) would be modified which assume a new corrections to the Newton's law of gravitation. Using the corrected entropy given in Eq. (4.19), we find that the number of bits should also be corrected as follows.

$$N' = \frac{4S}{k_B} = \frac{A}{\ell_p^2} + \frac{4}{3}\,\alpha_0\,\sqrt{\mu\,\pi\,\frac{A}{\ell_p^2}}. \tag{5.1}$$

By substituting Eq. (5.1) into Eq. (3.6) and using Eq. (3.1), we get

$$E = F\,c^2\left(\frac{r^2}{m\,G} + \frac{\alpha\,\sqrt{\mu}\,r}{3\,m\,G}\right). \tag{5.2}$$

It is apparent, that Eq. (5.2) implies a modification in the Newton's law of gravitation

$$F = G\frac{Mm}{r^2}\left(1 - \frac{\alpha\,\sqrt{\mu}}{3\,r}\right). \tag{5.3}$$

This equation states that the modification in Newton's law of gravity seems to agree with the predictions of Randall-Sundrum II model [35] which contains one uncompactified extra dimension and length scale Λ_R. The only difference is the sign. The modification in Newton's gravitational potential on brane [36] is given as

$$V_{RS} = \begin{cases} -G\frac{mM}{r}\left(1 + \frac{4\Lambda_R}{3\pi r}\right), & r \ll \Lambda_R \\ \\ -G\frac{mM}{r}\left(1 + \frac{2\Lambda_R}{3r^2}\right), & r \gg \Lambda_R \end{cases}, \tag{5.4}$$

where r and Λ_R are radius and the characteristic length scale, respectively. It is clear that the gravitational potential is modified at short distance. We notice that our result, Eq. (5.3), agrees with different sign with Eq. (5.4) when $r \ll \Lambda_R$. This result would say that $\alpha \sim \Lambda_R$ which would help to set a new upper bound on the value of the parameter

α. This means that the proposed GUP-approach [32, 34] is apparently able to predict the same physics as Randall-Sundrum II model. The latter assumes the existence of one extra dimension compactified on a circle whose upper and lower halves are identified. If the extra dimensions are accessible only to gravity and not to the standard model field, the bound on their size can be fixed by an experimental test of the Newton's law of gravitation, which has only been led down to ~ 4 millimeter. This was the result, about ten years ago [54]. In recent gravitational experiments, it is found that the Newtonian gravitational force, the $1/r^2$-law, seems to be maintained up to $\sim 0.13 - 0.16$ mm [55]. However, it is unknown whether this law is violated or not at sub-μm range. Further implications of this modifications have been discussed in [56] which could be the same for the GUP modification which is calculated in this paper. This similarly between the GUP implications and extra dimensions implications would assume a new bounds on the GUP parameter α with respect to the extra dimension length scale Λ_R.

6   Conclusions

In this paper, we tackle the consequences of the quantum gravity on the entropic force approach which assumes a new origin of the gravitational force. We found that the quantum gravity corrections lead to a modification in the area law of the entropy which leads to a modification in the number of bits N. According to Verlinde's theory of entropic force, the Newton's law of gravitation would acquire new quantum gravity corrections due to the modified number of bits. The modification in the Newton's Law of gravitation surprisingly agrees with the corrections predicted by Randall-Sundrum II model with different sign. This would open a new naturally arising question in our proposed research if the GUP and extra dimensions theories would predict the same physics. We hope to report on this in the future.

Acknowledgments

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